A Theory of Social Impact Bonds*

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Abstract

Social Impact Bonds (SIBs) are an innovative financing mechanism for public goods. In a SIB, an investor provides capital to a non-profit service provider for a social intervention. The investor receives a return based on the outcome of the intervention relative to a pre-determined benchmark. We describe the basic structure of a social impact bond and provide some descriptive statistics for these financial instruments. We then consider a formal model of social impact bonds and examine their ability to finance positive net present value projects that traditional debt finance can not. We find that SIBs expand the set of implementable projects if governments are pessimistic (relative to the private sector) about the probability an intervention succeeds or if the government is particularly adverse to states of the world where it must pay costs associated with a project without any offsetting benefits. As both these features are present in a variety of public programs, we conclude that SIBs are a real innovation in public finance and should be considered for projects where traditional debt finance has been rejected.

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1 Introduction

Impact investing, financial investing that values positive social or environmental impact as well as traditional financial returns, is a growing field in asset management.[1] In 2018 $12 trillion dollars of assets under management were managed by professionals who considered environmental and social factors in their investment choices, representing a 38% increase over the previous two years [Aitken et al. (2018)]. One type of impact investing is the social impact bond (SIB), an arrangement where investors in a project receive financial returns based on the project outcomes, specifically the accomplishment of pre-specified social objectives.

In a traditional social impact bond, a government attracts financing from an investor. The investor passes on the financing to a service provider who implements a project designed to solve a specific social problem. If the project meets specified social goals, e.g. test scores for a education intervention, the investor receives the initial investment back plus interest payments. In many of these arrangements, investors do not receive any money back if the goals are not met. In order for payments to be made, a third-party assessor must certify that the intervention achieved the pre-specified goals. Given the complexity of these arrangements, it is common for an additional entity, known as the intermediary, to co-ordinate the various parties to the social impact bond arrangement.[2]

Social impact bonds have been growing since their initial launch in September of 2010. For example 2017 saw the launch of 41 unique social impact bonds. There are currently 81 social impact bond projects with a total of $375 million dollars invested.[3]

Despite the growth of social impact bonds as a vehicle for financing public programs, with the exception of Pauly and Swanson (2017), we have been unable to find a theoretical analysis of the benefits of social impact bonds. Our paper attempts to fill this gap. Specifically, with

[1] For academic work on impact investing see Bialkowski and Starks (2016) or Renneboog et al. (2008) for a survey.

[2] Many reports exist describing the structure of social impact bonds. For one example see Gustafsson-Wright et al. (2015)

[3] Statistics are author's calculations based on data from the Social Finance Impact Bond Global Database. https://sibdatabase.socialfinance.org.uk/. See Social Finance (2019)

in the context of an economic model, we compare social impact bonds to debt finance and characterize the conditions under which social impact bonds can expand the set of projects which a government can finance.

We find that social impact bonds have advantages over traditional debt finance. Specifically, when the government is overly pessimistic or particularly adverse to states of the world where project costs can not be offset with project benefits, then social impact bonds can finance positive net present value projects that debt finance cannot. We conclude that social impact bonds are a useful form of finance and should be considered for projects where debt finance has been rejected.

The intuition for our main results are straightforward. First, when governments are pessimistic about the probability a project will be successful, a social impact bond that pays off only when a project is successful will have low ex ante expected financing costs. On the other hand, an optimistic private sector will view this bond as a good investment opportunity. Second, when the government is adverse to states of the world where the project fails and it must make interest payments associated with the project, it will prefer a financing arrangement that has it make larger payments when the project is successful in exchange for lower payments if the project fails.

The theoretical literature on social impact bonds is remarkably thin. To date, we have found just one paper [Pauly and Swanson (2017)] that contains a model of social impact bonds. Like their paper, our paper contains an equivalency result, i.e. debt finance and social impact bonds are equivalent in a world with perfect information and no risk aversion. However, we differ from Pauly and Swanson in how we break the equivalency. Specifically, they consider models where effort is unobservable and SIBs can incentivize effort. We consider models where there is asymmetric information about the probability of project success and governments, because of electoral concerns, are particularly adverse to states of the world where they must pay cost associated with a project in excess of benefits.

This equivalency approach links to the theoretical literature on corporate social responsi-

bility (CSR). There, Small and Zivin (2005) argue that firms engaged in CSR and those that are not will be priced the same when firm provided public good are substitutes for non-profit provided services. In contrast, Baron (2008) allows for unobservable effort and a consumer and manager affinity for providing public goods and demonstrates that some level of CSR is optimal for the firm. However, SIBs differ from CSR. With CSR the firm is providing a public good. With a SIB, a for-profit firm provides financing, but the service is carried out by a not-for-profit entity.

A key mechanism in our model is a desire for governments to avoid states of the world where they must pay costs associated with a project (e.g. financing costs) but receive no benefits from the project (e.g. a failed educational initiative). This mechanism relates to the literature on optimal debt management [e.g. IMF and the World Bank Staff (2001); Missale (1997); Filardo et al. (2012); Vajs (2014)] that stresses the need to incorporate long-term debt in the country portfolios to avoid the risk of funding crises. While we do not consider projects large enough that failure could lead to a public debt crisis, we show how careful structuring of a SIB can reduce the government's risk exposure.

Interestingly, this approach paralles the static theory of optimal corporate capital structure, e.g Scott Jr (1976). In this theory, firms issue equity despite its tax disadvantages (see Miller (1977)) to lower the probability of financial distress.[4] Similarly, to avoid high debt payments in bad states of the world a government would wish to issue equity. However, the government can not issue ownership shares. However, SIBs can provide an equity like instrument that allows interest payments to depend on the state of the world. The benefit of conditioning interest payments on the state of the world is also stressed in the literature on state contingent sovereign debt, which argues that governments should issues bonds where interest payments depend on the state of the macroeconomy [IMF and the World Bank Staff (2017)].

Finally, our work relates to the theoretical literature on public-private partnerships:

[4]See also: Fischer et al. (1989); Hennessy and Whited (2005); Marsh (1982); Stein (1992).

Maskin and Tirole (2008); Iossa and Martimort (2015); Hart (2003); Reich (2000). However, there is a key difference. With a public-private partnership, the government contracts with a for-profit firm to provide a service. With a SIB, a non-profit firm provides the service, and the for-profit firm provides the financing.

The next section provides background on social impact bonds and some descriptive statistics. Section 3 describes the main model of the paper. Section 4 derives and discusses the key insights of our SIB. Section 5 concludes.

2 Background on social impact bonds

A social impact bond is "a multi-stakeholder partnership in which private investors, philanthropic funders and impact investors—not governments—take on the financial risk of expanding preventive programs that help poor and vulnerable people," Callanan et al. (2000) All social impact bonds require four core agents to function: the issuer, investors, service providers, and a third-party assessor. The issuer is the entity that issues the social impact bond. The Issuer is most often a public entity, i.e. the government, so much so that hereafter we will use "issuer" and "government" interchangeably. In traditional public financing of social programs, the government provides up-front funding to service providers and shoulders the financial risk associated with the funded program. Under a SIB arrangement, private investors provide the up-front capital and the government agrees to repay them for the program's cost (plus an agreed-upon rate of return) if it achieves predefined outcomes. This allows the government to (1) pay only for successful programs, (2) pay with a portion of the realized benefits from the program (e.g. cost-savings) and (3) align payment with the realization of benefits. Private investors provide up-front funding for the administration, execution, and evaluation of the program. Investors accept the risks associated with program failure in exchange for a commensurately higher rate of return than that of sovereign debt. Private investors could include impact investors (both small- and large-scale) and

philanthropic individuals or organizations. [5]

2.1 Examples

2.1.1 Peterborough Recidivism Impact Bond

The world's first social impact bond, the Peterborough Recidivism Impact Bond, was pioneered in the UK in 2010 as part of an effort by the Ministry of Justice to reduce reconviction rates among non-violent offenders [Disley et al. (2015)]. The bond funded work by the One Service to operate at Peterborough Prison and provide three cohorts of 1,000 male prisoners with pre- and post-release support in housing, training and employment, parenting, substance abuse, and mental health. Social Finance, a nonprofit organization, acted as the intermediary to coordinate the project and raised capital from a total of 17 investors (principally charitable trusts and foundations) who would receive outcome payments if reconviction rates fell by an average of 7.5% or more across the targeted cohorts.

The Peterborough Impact bond was highly successful on several fronts. First, reconviction rates fell by an average of 9% across the cohorts, triggering repayment in full for all investors plus a return of 3% interest per annum.[6] Second, the program led to sustained, ongoing services that continued to operate at Peterborough after the end of the pilot. Third, assessments of the program reported that interviewees found the SIB funding, with its focus on outcomes-based payments, to be more flexible and responsive to feedback from program monitors.

[5] See among other Gustafsson-Wright et al. (2015); Lower-Basch (2014) for descriptions of the functioning of social impact bonds.

[6] Originally intended to operate until 2017, the Peterborough Impact Bond was ended early in 2015 to avoid any duplication in services after roll-out of the national-wide Transforming Rehabilitation program that mandated statutory supervision for short-sentenced offenders. Outcome payments were ultimately based on average recidivism rate reduction across the first two cohorts (9%).

2.1.2 Educate Girls Social Impact Bond

Over the past decade, social impact bonds have since been used in a wide array of other contexts that highlight their versatility and customizability. For example, the Educate Girls Development Impact Bond was the first impact bond to be launched in a developing country context in 2015. Operating in Rajasthan, India, the project aimed to address issues of educational access and attendance among schoolgirls. In rural parts of the state (and at the time of the launch), a girl was twice as likely to be out of school as a boy and only 50% of women could read or write. The impact bond provided funding to the NGO, Educate Girls, to develop more targeted curriculums and improve outreach to 9,000 unenrolled schoolgirls living in marginalized areas. Whereas the Peterborough bond's payment thresholds were based on a single measure (recidivism reduction rates), payments for the Educate Girls boned were structured around two performance measures: learning levels (which determined 80% of the outcome payment) and enrollment rates (which determined the remaining 20%).

The Educate Girls bond saw even greater success than that of the Peterborough Bond: the program achieved 160% of its target learning outcomes threshold (measured by standardized testing results), and by the end of the third year, enrollment rates had reached 92%, far surpassing the target 79%. Investors were repaid their principal investments in full and received an impressive 15% return, which is set to be reinvested in future development programs.[7]

2.1.3 Cameroon Cataract Development Bond

More recently, a currently ongoing social impact bond focused on ophthalmic health was issued in Cameroon in 2017.[8] The vast majority of blindness and vision impairment cases in Sub-Saharan Africa are preventable or treatable, half of which are related to cataracts and

[7]Results for the Educate Girl's social impact bond are obtained from Foundation (2018) available at: https://instiglio.org/educategirlsdib/about-the-dib/

[8]Oroxom et al. (2018) provide a detailed description of the Cameroon bond. Their paper is available at :https://golab.bsg.ox.ac.uk/knowledge-bank/case-studies/cameroon-cataract-bond/

can be corrected through a simple, 20-minute surgery. The Cameroon Cataract Development Bond creates a financing vehicle to fund as many as 18,000 cataract surgeries over five years. This bond structures its payment thresholds around three outcome categories: volume of completed surgeries (7,000 by year three and 18,000 by year 5), quality of completed surgeries (50% or higher success rate), and sustainability of the hospital. This bond well illustrates the flexibility of the structure of social impact bond: it is unique in having a coalition of three outcomes funders, none of which are government entities, and offering an added performance incentive for the service provider of $120,000 for equity in provision.[9] Scheduled to conclude in 2023, it is currently on target to meet all three outcomes goals, and the potential returns to investors (8%) are high relative to traditional sovereign debt. Not all social impact bonds have seen such successful outcomes. The United States' first social impact bond.

2.1.4 The ABLE Project for Incarcerated Youth

The ABLE Project for Incarcerated Youth was established in 2013 with the goal of reducing recidivism among incarcerated youth at Riker's Island jail. At the program's inception, almost 50% of 18-year-old men released from Riker's returned to jail within two years. The intervention provided young men aged 16 through 21 job training, transitional employment and job placement services. A successful outcome was defined as a 10% reduction in the number of days participants were jailed during the subsequent year. Preliminary results found no statistically significant reduction in recidivism among those who received the intervention. Therefore, the program shut down early and investors were not repaid. (Citation?)

[9]These outcomes funders include the Conrad N. Hilton Foundation, The Fred Hollows Foundation, and Sightsavers.

3 Data and Descriptive Statistics

3.1 Data Description

3.2 Descriptive Statistics

Table 1 provides descriptive statistics for completed and ongoing social impact bond projects. The vast majority (95%) of social impact bond projects are located in high-income countries, with the highest concentration in Europe and Central Asia (56%) followed by North America (23%) and East Asia and the Pacific (12%) (Table 1). There have been only three social impact bond projects in Latin America (located in Peru, Argentina, and Colombia), four in Sub-Saharan African (located across South Africa, Cameroon, Uganda, Kenya, the Congo, Mali, and Nigeria), two in the Middle East (both in Israel), and three in South Asia (all in India). These projects cover a broad set of issues areas, from affordable housing programs to medical procedures to sanitation infrastructure to wildlife conservation programs. Most projects, however, fall under the umbrellas of workforce development, child and family welfare, or housing and homelessness. One possible explanation for such concentration in these issue areas is that they often involve shorter-term intervention programs. Programs with shorter durations may lend themselves to earlier evaluation and faster repayment for investors.

Following their initial launch with the Peterborough Impact Bond in 2010, new social impact bond projects began gaining traction. The number of new bonds peaked in 2017, with 51 launching in that year alone. 2018 saw a drop to 11 new bonds – the lowest since 2014. Additionally, 2019 launches to date appear consistent with this decline. It is unclear whether the drop in recent years will be indicative of future trends in the creation of new social impact bond. The first health-focused social impact bond launched in 2015. Health became an increasingly popular issue area for social impact bonds over the next two years, increasing from 2 new launches in 2015 to 12 in 2017. Though the number of launches, including

Table 1: Social Impact Bond Project Distribution by Issue Area, Country Income Classification, and Region

Project Characteristic	**Count**	**Percent**
Issue Area		
Workforce Development	41	31%
Child and Family Welfare	20	15%
Housing/Homelessness	22	17%
Health	22	17%
Criminal Justice	12	9%
Education and Early Years	11	8%
Poverty and Environment	3	2%
Country Income Classification		
High income	122	95%
Upper middle income	3	2%
Lower middle income	4	3%
Low income	0	0%
Region		
Europe and Central Asia	73	56%
North America	30	23%
East Asia and Pacific	16	12%
Sub-Saharan Africa	4	3%
Latin America and the Caribbean	3	2%
South Asia	3	2%
Middle East and North Africa	2	2%

Source: Social Finance Impact Bond Global Database and Authors' research. https://sibdatabase.socialfinance.org.uk/. Notes: 1. Excludes two projects undertaken in multiple countries of different income classifications. This includes work in one low-income country, Uganda, which is not reflected in the above aggregation.

health-focused launches, declined sharply in 2018, health-focused launches comprised 27% of all new social impact bonds – the highest percentage to date.

Total investment in social impact bonds has increased almost every year since their inception in 2010 with the Peterborough Impact Bond (in 2011 there were no new bonds). 2012-2017 saw steady increases in total investment, with new investment peaking in 2016. Despite the fact that the growth rate of investment levels has slowed in recent years, total global investment in social impact bonds is the highest it has ever been at 403 million USD. This is an increase in total global investment of 1.5 times the new investment peak in 2016.

Figure 1: SIB Projects by Launch Year and Issue Area 2010 - 2019

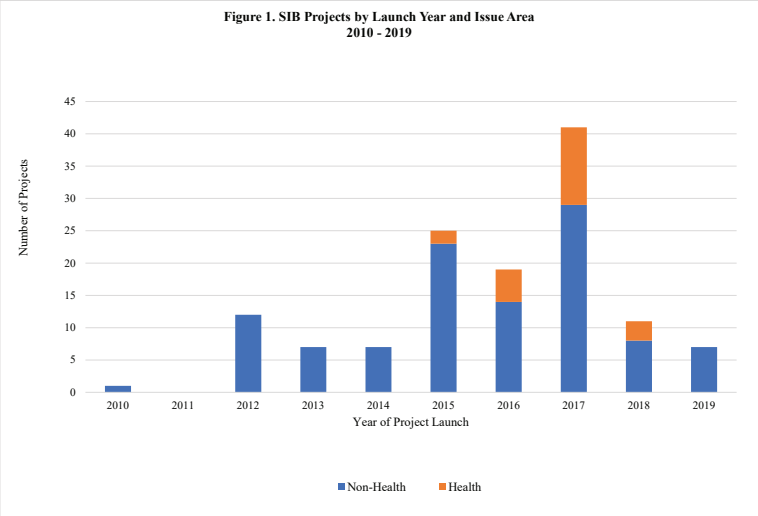
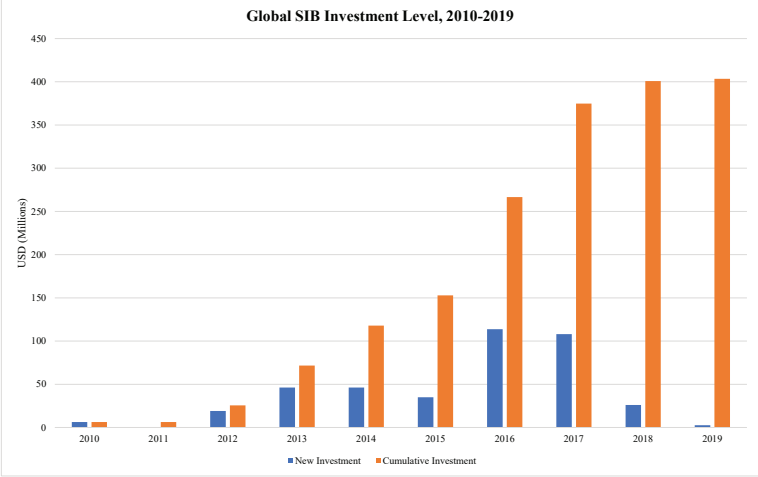


Figure 2: Global SIB Investment Level, 2010-2019



Description of table 2 and caption

Table 2: Investment Data

Country	Capital Raised (USD Millions)	Max Outcomes Payments (USD Millions)	Number of Investors	Time to Maturity (Years)
UK	1.64	3.59	3	3.97
	(0.37 6.10)	(0.10 12.2)	(1 17)	(2 10)
US	8.77	9.95	5	6.56
	(0.15 30)	(0.15 34)	(1 40)	(3 30)
Other	2.9	4.32	4	4.3
	(0.11 27)	(0.23 15.81)	(1 59)	(1 10)

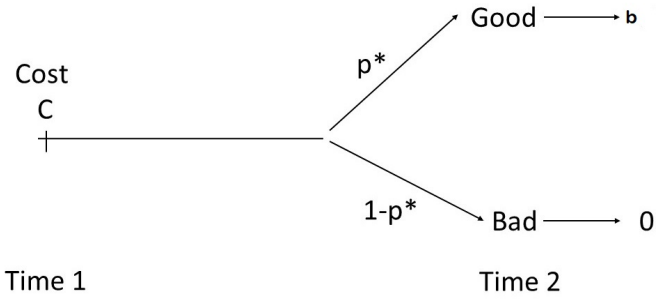
4 Model

This section contains our model of social impact bonds. It first describes a project the government could potentially finance. Next, it describes the objectives of the government and the investor who may finance the project. Then we define the financing instruments: a debt contract and a social impact bond. Finally, we describe the incentive compatibility conditions which are required for the government to provide payment and the investor to provide financing sufficient to implement the project.

4.1 Project

There is a project which the government is deciding to finance. The project costs c to implement this period and has a potential benefit of b next period. Next period there are two possible states of the world, the good state and the bad state. In the good state the government receives the benefit b , in the bad state the government receives zero. The good state of the world occurs with unknown probability denoted by p^*. The figure below summarizes the project.

Figure 3: Project



4.2 Financing

There are two possible mechanisms to finance the project. The first, debt finance, constrains the government's payments to be constant across the two states. The second, a social impact bond, allows the payments to be conditional on the benefits the government receives and therefore the state of the world.

4.3 Agents

The model contains two types of agents: a risk-neutral investor and the government. The risk-neutral investor will only invest in a project if she expects to get a return greater than or equal to her required return. Her required return is given by:

$$r - \omega. \tag{1}$$

Here r is the expected return on comparable government debt, ω represents the discount an investor is willing to take to invest in a socially responsible project. The investor also believes that the project will succeed with probability p, which may or may not equal p^* (the true probability).

The second type of agent is the government. The government maximizes the expected present value of benefits minus costs from implementing the project. However, the government has an added preference for avoiding unfunded costs. They are specifically adverse

to experiencing states of the world where they must pay costs associated with a project (e.g. debt payments) when there are no corresponding benefits. While we do not currently model why this is the case, one potential explanation is failed projects can be portrayed as wasteful spending by political opponents reducing a politician's reelection probability. The magnitude of this aversion is captured with the parameter ϕ.

Subsequently, the government's expected utility is given by:

$$E(U) = -(1+\phi)c_1 + \frac{1}{1+r}\left[q\left(b - c_2^g\right) - (1-q)c_2^b\right] - \frac{\phi}{1+r}\left[q * max(0, c_2^g - b) + (1-q) * max(0, c_2^b)\right] \tag{2}$$

In this function c_1 are any costs the government pays in the first period, c_2^g are costs the government pays in the second period if the good state of the world occurs, c_2^b are costs the government pays in the second period if that bad state of the world occurs, $\frac{1}{1+r}$ is the government's discount factor, and q is the government's belief for the probability that good state of the world will occur. Like the investor, this probability q may not necessarily equal the true probability p^*. Finally, note that when the government chooses to not finance the project, it's utility equals zero.

To grasp this utility function note that the first term captures government costs in the first period. The cost, c_1 is multiplied by $1 + \phi$ because there are no benefits in the first period to offset any costs and therefore, given the government's aversion to unfunded costs, government utility falls by an additional factor ϕ. The second term represents the expected, present discounted value of net benefits from the project. The third term represents the expected, present discounted value of unfunded benefits. The use of the max function ensures that when benefits b exceed costs in the good state of the world c_2^g the unfunded benefit is exactly zero.

4.4 Financial Instruments

There are available instruments to finance the project: traditional debt finance and a social impact bond.

Definition 1. *Debt finance* is a contract that requires the government to pay the investor $c(1+i)$ in each state of the world. i is the interest rate on the debt contract.

Definition 2. A *social impact bond* (SIB) is a contract that requires the government pay the investor pre-specified values $c_2^g \geq 0$ in the good state of the world and $c_2^b \geq 0$ in the bad state of the world.

4.5 Incentive Compatibility

A project is *government incentive compatible* if the government would choose to implement the project with an available financing instrument. The project is *investor incentive compatible* if the investor is willing to provide financing for the project. For the investor, incentive compatibility requires that her expected return be greater than the required return. For the government, incentive compatibility requires that the expected utility of implementing the project is greater than the utility of not implementing the project, 0.

Definition 3. A project is *implementable* if it is incentive compatible for both the investor and the government.

5 Results

This section examines the conditions under which the government can implement potential projects with debt finance and the conditions under which a social impact bond can expand the set of projects that can be financed.

5.1 Debt finance

Theorem 1. *A debt contract with interest rate i can implement the project if and only if* $\frac{qb}{c[1+\phi(1-q)]} - 1 \geq r - \omega$.

Proof. See appendix. □

To understand this result, note that the condition requires that the government's expected return on the project be greater than the required return of the investor. However, note that the expected cost of the project to the government is not just c but $c[1+\phi(1-q)]$. This is the government's real cost because, based on its beliefs, with probability $1-q$ the government will have to make debt payments in excess of received benefits which has an additional cost of ϕ per dollar of unfunded costs. However, when the expected benefit of the project is high enough to cover the government's interest costs taking into account its aversion to unfunded costs, the government is willing to finance the project with debt.

Remark 1. When the government has no preference for avoiding states of the world with costs and no benefits, i.e. $\phi = 0$, then any project where $\frac{qb}{c} - 1 \geq r - \omega$, can be implemented. This result is the conventional result that the government will finance any project where the expected return is greater than the rate at which it can borrow.

Remark 2. When $\frac{qb}{c[1+\phi(1-q)]} - 1 > r - \omega$, any interest rate i where $\frac{qb}{c[1+\phi(1-q)]} - 1 > i > r - \omega$ can implement the project. The higher the interest rate the larger the share of the expected benefit from the project that is paid out to the investor.

5.2 Social Impact Bond

Now we consider the set of projects that can be implemented under a social impact bond and specifically the capacity of a social impact bond to expand the set of implementable projects.

Theorem 2. *A social impact bond is investor incentive compatible if and only if* $\frac{pc_2^g + (1-p)c_2^b}{c} - 1 > r - \omega$ *and is government incentive compatible if and only if* $q\frac{b}{c} > \frac{qc_2^g + (1-q)(1+\phi)c_2^b}{c}$.

Proof. See appendix. □

Theorem 3. *A project is implementable using a social impact bond whenever* $p\frac{b}{c} > (r-\omega+1)$. *Moreover if* $p \geq q$, *this condition will hold for any project that is implementable with a social impact bond.*

Proof. See appendix □

To develop some intuition for this result note that if the investor believes the good state of the world will occur with probability p , the left hand side of this condition is the expected return on the contract that pays all the project benefits to the investor in the good state and pays zero in the bad state. The government would accept that contract because it would set government utility to exactly zero. Therefore, whever this condition holds we can implement the project.

To understand why when $p \geq q$ this condition must hold for any implementable project consider the contract $c_2^g = \frac{c(r-\omega+1)}{p}$ and $c_2^b = 0$. This contract has the benefit of completely eliminating unfunded costs and exactly compensating the investor by giving her exactly the required return. Moreover when $p \geq q$ the investor values payments in the good state relatively more than the government because the investor views these states as more likely. As such it is best for the government to pay as much as possible in the good state. Therefore from the government's point of view this is the best contract and if it can not implement the social impact bond no other contract will. Now note, the government would accept this contract if it gives a positive expected return to the government. The government will get a positive expeced return if $q(b - \frac{c(r-\omega+1)}{p}) > 0$ which leads to the above condition.

An important implication of theorem 3 is that when there is an investor that is at least as optimistic as the government any project with positive expected value from the point of view of the government can be implemented.

With only debt finance theorem 1 demonstrates that only projects where the expected return $\frac{qb}{c} \geq (1 + r - \omega) * ([1 + \phi(1 - q)]$ will be implementable. We can think about the

government's aversion to unfunded costs as creating a wedge between the expected return on a project and the cost of debt finance. The size of the wedge is proportional to ϕ , the magnitude of the government's aversion to unfunded costs, and to $1-q$, the probability the government experiences unfunded costs.

Since a social impact bond can eliminate these unfunded costs, if the government can find an investor that is at least as optimistic as it is, it can fund any project with an expected value greater than the interest cost of debt, i.e. $\frac{qb}{c} \geq (1+r-\omega)$ since there is an investor with a p where $\frac{pb}{c} \geq (1+r-\omega)$. As such, the social impact bond provides a solution to the inefficiency caused by the government's aversion to unfunded costs.

Remark 3. When $\phi = 0$ and $p \leq q$ any project which is implementable with a social impact bond is also implementable with debt finance.

Proof. Setting $\phi = 0$ in theorem 2 we obtain that a social impact bond is only incentive compatible if government incentive compatible $q\frac{b}{c} > \frac{qc_2^g+(1-q)c_2^b}{c}$. Recall, also from theorem 2, that the social impact bond is only investor incentive compatible if $\frac{pc_2^g+(1-p)c_2^b}{c} - 1 > r - \omega$. When $q \geq p$ these two conditions imply $\frac{qb}{c} - 1 \geq r - \omega$ which is the implementability condition from theorem 1 for debt finance. □

Remark 3 implies that for social impact bonds to add value by increasing the set of implementable projects, we must either have investors that are more optimistic than the government or have a government that is adverse to unfunded costs.

6 Conclusion

In this paper we formulated a novel theory of social impact bonds. Social impact bonds allow the government to finance positive net present value projects that can not be financed with tradition debt finance. SIBs solve two inefficiencies. The first inefficiency arises when the government is unduly pessimistic about the possible success of the project. The inefficiency

arises when political realities lead politicians to be overly concerned about paying for unsuccessful projects. By allowing the payouts to depend on the project's success and the beliefs of the investor the social impact bond can implement positive net present value projects when one or both of these inefficiencies prevent the project from being financed through traditional debt finance. As both these features are common in public investment, SIBs represent a useful innovation in public finance and should be considered when traditional debt financing has been rejected.

Looking forward, their are additional features of social impact bonds that have yet to be explored. One important feature is the role and investor can play in affecting the probability a project succeeds. Another important issue to explore is the SIB transaction cost relative to traditional debt finance and the resulting appropriate scale of social impact bonds.

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A Proof of theorems

Theorem 1. *A debt contract with interest rate i can implement the project if and only if* $\frac{qb}{c[1+\phi(1-q)]} - 1 \geq r - \omega$.

Proof. If a project is incentive compatible for the government it must by true that $b > c_2^g$ otherwise every term in the government's utility would be negative. Given that $b > c_2^g$, the government's expected utility is given by $\frac{1}{1+r}\left[q\left(b - c_2^g\right) - (1-q)c_2^b\right] - \frac{\phi}{1+r}\left[(1-q)c_2^b\right]$. Substituting in $c(1+i) = c_2^g = c_2^b$ we have that the government's expected utility equals $\frac{1}{1+r}\left[b - c(1+i)\right] - \frac{\phi}{1+r}\left[(1-q)c(1+i)\right]$which is greater than zero if and only if $\frac{b}{c[1+\phi(1-q)]} - 1 \geq i$. Since the project is incentive compatible for the investor if and only if $i \geq r - \omega$. The project is implementable if and only if $\frac{b}{c[1+\phi(1-q)]} - 1 \geq r - \omega$ □

Theorem 2. *A social impact bond is investor incentive compatible if and only if* $\frac{pc_2^g + (1-p)c_2^b}{c} - 1 > r - \omega$ *and is government incentive compatible if and only if* $q\frac{b}{c} > \frac{qc_2^g + (1-q)(1+\phi)c_2^b}{c}$.

Proof. Since the social impact bond pays the investor c_2^g in the good state and c_2^b in the bad state of the world, and the investor believes the good state will occur with probability p then her expected return is equal to $\frac{pc_2^g + (1-p)c_2^b}{c}$ for the initial investment of c. This is greater than the required return if and only if $\frac{pc_2^g + (1-p)c_2^b}{c} > 1 + r - \omega$.

If a project is incentive compatible for the government it must by true that $b > c_2^g$ otherwise every term in the governments utility would be negative. Given that $b > c_2^g$, the government's expected utility is given by $\frac{1}{1+r}\left[q\left(b - c_2^g\right) - (1-q)c_2^b\right] - \frac{\phi}{1+r}\left[(1-q)c_2^b\right]$. Utility is non-negative when $qb - qc_2^g - (1-q)c_2^b - \phi(1-q)c_2^b > 0$. Rearranging we get $q\frac{b}{c} > \frac{qc_2^g + (1-q)(1+\phi)c_2^b}{c}$.

See appendix □

Theorem 3. *A project is implementable using a social impact bond whenever* $p\frac{b}{c} > (r - \omega + 1)$. *Moreover if $p \geq q$, this condition will hold for any project that is implementable with a social impact bond.*

Proof. Based on the previous theorem a project is government incentive compatible if and only if $q\frac{b}{c} > \frac{qc_2^g+(1-q)(1+\phi)c_2^b}{c}$. To find the maximum set of projects that will satisfy this condition we consider the social impact bond contract that makes the investor indifferent between investing and not investing in the social impact bond, i.e. her incentive compatibility constraints holds as an equality. Therefore, $pc_2^{g^*} + (1-p)c_2^{b^*} = c(r-\omega+1)$ which implies $c_2^{b^*} = \frac{c(r-\omega+1)-pc_2^{g^*}}{1-p}$. From the previous theorem, this contract is incentive compatible for the government if and only if $q\frac{b}{c} > q\frac{c_2^{g^*}}{c} + \frac{(1-q)(1+\phi)}{c}\left[\frac{c(r-\omega+1)-pc_2^{g^*}}{1-p}\right]$ or $q\frac{b}{c} > \left[q - \frac{p(1-q)(1+\phi)}{1-p}\right]\frac{c_2^{g^*}}{c} + \frac{(1-q)(1+\phi)}{1-p}(r-\omega+1)$.

Consider the contract $c_2^{g^*} = \frac{c(r-\omega+1)}{p}$ and $c_2^{b^*} = 0$. For this contract the government's implementability constraint simplifies to $q\frac{b}{c} > q\frac{c(r-\omega+1)}{p}$ or $p\frac{b}{c} > (r-\omega+1)$. Therefore when this condition holds there is a social impact bond that can implement the project.

Additionally when $p \geq q$, the first term on the right hand side of the government's incentive compatibility condition $\left[q - \frac{p(1-q)(1+\phi)}{1-p}\right]\frac{c_2^{g^*}}{c}$is negative. Since we will maximize the set of implementable projects by making the right hand side of the government's compatibility constraint as small as possible, we can maximize the set of implementable projects with $c_2^{g^*} = \frac{c(r-\omega+1)}{p}$ and $c_2^{b^*} = 0$. In this case the implementability constraint simplifies to $q\frac{b}{c} > q\frac{c(r-\omega+1)}{p}$ or $p\frac{b}{c} > (r-\omega+1)$. moreover, our contract that implements the social impact bond maximizes the set of implementable projects. Therfore this condition described all implementable projects. □